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Math 320
Lia Vas

Modeling with First Order Differential Equations

In most cases, it is equally important to be able to come up with a differential equation that accurately describes the problem you need to solve as to being able to solve the equation. The process of writing an equation describing the situation is called the mathematical modeling. In order to successfully model a problem with and solve a differential equation, it might be helpful to ask yourself the questions listed below.

1. **Identify the real problem. Identify the problem variables.** What do we need to find out? What is the problem asking for?
2. **Construct appropriate relation between the variables – a differential equation** What is dependent, what independent variable and what is the rate of change? Figuring out how these quantities are related will result in a differential equation that models the problem.
3. **Obtain the mathematical solution.** Recognize the type of the equation. Decide if you can solve it analytically ("by hand") or if you need to use the technology. In both cases, decide on the method that you will use (e.g. is the equation separable, linear or some other type; could Euler's method, ODE45 or other numerical solution be found).
4. **Interpret the mathematical solution.** After solving the equation, check if the mathematical answer agrees with the context of the original problem. **Check the validity**: Does your answer make sense? Do the predictions agree with real data? Do the values have correct sign? Correct units? Correct size? **Check effectiveness**: Could a simpler model be used? Have I found a right balance between greater precision (i.e. greater complexity) and simplicity?

Example 1. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. Find the number of bacteria after 4 hours.

Discussion. Identifying variables: let y stands for the bacteria culture and t stands for time passed. The first part of the problem "A bacteria culture starts with 500 bacteria..." tells us that $y(0) = 500$. The second part "... and grows at a rate proportional to its size." is the key for getting the mathematical model. Recall that the rate is the derivative and that "...is proportional to..." corresponds to "equal to constant multiple of." So, the equation relating the variables is $\frac{dy}{dt} = ky$. The solution of this differential equation is $y = y_0 e^{kt}$. Since $y_0 = 500$, it remains to determine the proportionality constant k. From the condition "After 3 hours there are 8000 bacteria." we obtain that $8000 = 500e^{3k}$ which gives us that $k = \frac{1}{3}\ln 16 = .924$. Thus, the number of bacteria after t hours can be described by $y = 500e^{.924t}$.

Solution. Using the function we have obtained, we find the number of bacteria after 4 hours to be $y(4) = 20159$ bacteria.

Example 2. Suppose that an object is falling in the atmosphere near the sea level. Assume that the drag is proportional to the velocity with the drag coefficient of 2 kg/sec and that the mass of the object is 10 kg.

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